OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4763

Mechanics 3

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

1 **(a)** **(i)** Write down the dimensions of force. [1]

The period, t, of a vibrating wire depends on its tension, F, its length, l, and its mass per unit length, σ.

(ii) Assuming that the relationship is of the form $t = kF^{\alpha}l^{\beta}\sigma^{\gamma}$, where k is a dimensionless constant, use dimensional analysis to determine the values of α, β and γ. [6]

Two lengths are cut from a reel of uniform wire. The first has length 1.2 m, and it vibrates under a tension of 90 N. The second has length 2.0 m, and it vibrates with the same period as the first wire.

(iii) Find the tension in the second wire. (You may assume that changing the tension does not significantly change the mass per unit length.) [4]

(b) The midpoint M of a vibrating wire is moving in simple harmonic motion in a straight line, with amplitude 0.018 m and period 0.01 s.

(i) Find the maximum speed of M. [3]

(ii) Find the distance of M from the centre of the motion when its speed is 8 m s^{-1}. [4]

2 **(a)** A moon of mass 7.5×10^{22} kg moves round a planet in a circular path of radius 3.8×10^{8} m, completing one orbit in a time of 2.4×10^{6} s. Find the force acting on the moon. [4]

(b) Fig. 2 shows a fixed solid sphere with centre O and radius 4 m. Its surface is smooth. The point A on the surface of the sphere is 3.5 m vertically above the level of O. A particle P of mass 0.2 kg is placed on the surface at A and is released from rest. In the subsequent motion, when OP makes an angle θ with the horizontal and P is still on the surface of the sphere, the speed of P is $v\,\text{m s}^{-1}$ and the normal reaction acting on P is R N.

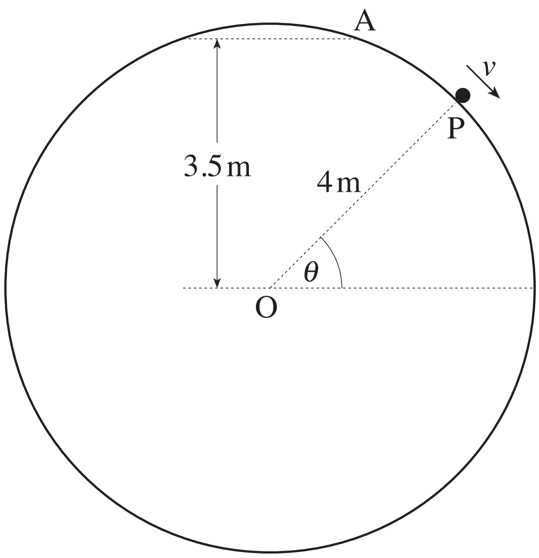

Fig. 2

(i) Express v^2 in terms of θ. [3]

(ii) Show that $R = 5.88 \sin\theta - 3.43$. [4]

(iii) Find the radial and tangential components of the acceleration of P when $\theta = 40°$. [4]

(iv) Find the value of θ at the instant when P leaves the surface of the sphere. [3]

3 A light elastic rope has natural length 15 m. One end of the rope is attached to a fixed point O and the other end is attached to a small rock of mass 12 kg.

When the rock is hanging in equilibrium vertically below O, the length of the rope is 15.8 m.

(i) Show that the modulus of elasticity of the rope is 2205 N. [2]

The rock is pulled down to the point 20 m vertically below O, and is released from rest in this position. It moves upwards, and comes to rest instantaneously, with the rope slack, at the point A.

(ii) Find the acceleration of the rock immediately after it is released. [3]

(iii) Use an energy method to find the distance OA. [5]

At time t seconds after release, the rope is still taut and the displacement of the rock *below the equilibrium position* is x metres.

(iv) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -12.25x$. [4]

(v) Write down an expression for x in terms of t, and hence find the time between releasing the rock and the rope becoming slack. [4]

4 The region between the curve $y = 4 - x^2$ and the x-axis, from $x = 0$ to $x = 2$, is occupied by a uniform lamina. The units of the axes are metres.

(i) Show that the coordinates of the centre of mass of this lamina are $(0.75, 1.6)$. [9]

This lamina and another exactly like it are attached to a uniform rod PQ, of mass 12 kg and length 8 m, to form a rigid body as shown in Fig. 4. Each lamina has mass 6.5 kg. The ends of the rod are at P$(-4, 0)$ and Q$(4, 0)$. The rigid body lies entirely in the (x, y) plane.

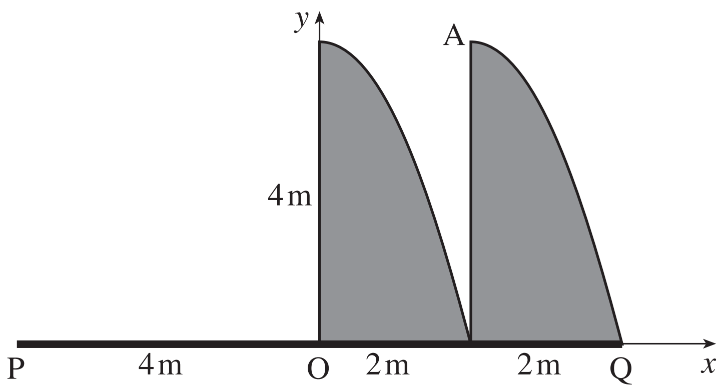

Fig. 4

(ii) Find the coordinates of the centre of mass of the rigid body. [5]

The rigid body is freely suspended from the point A$(2, 4)$ and hangs in equilibrium.

(iii) Find the angle that PQ makes with the horizontal. [4]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4763

Mechanics 3

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

1 **(a)** **(i)** Find the dimensions of power. [3]

In a particle accelerator operating at power P, a charged sphere of radius r and density ρ has its speed increased from u to $2u$ over a distance x. A student derives the formula

$$x = \frac{28\pi r^3 u^2 \rho}{9P}.$$

(ii) Show that this formula is not dimensionally consistent. [5]

(iii) Given that there is only one error in this formula for x, obtain the correct formula. [3]

(b) A light elastic string, with natural length 1.6 m and stiffness 150 N m^{-1}, is stretched between fixed points A and B which are 2.4 m apart on a smooth horizontal surface.

(i) Find the energy stored in the string. [2]

A particle is attached to the mid-point of the string. The particle is given a horizontal velocity of 10 m s^{-1} perpendicular to AB (see Fig. 1.1), and it comes instantaneously to rest after travelling a distance of 0.9 m (see Fig. 1.2).

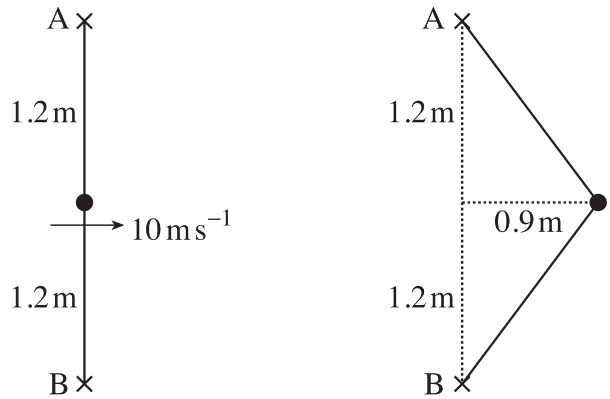

Fig. 1.1 **Fig. 1.2**

(ii) Find the mass of the particle. [5]

2 **(a)** A particle P of mass 0.6 kg is connected to a fixed point by a light inextensible string of length 2.8 m. The particle P moves in a horizontal circle as a conical pendulum, with the string making a constant angle of 55° with the vertical.

(i) Find the tension in the string. [2]

(ii) Find the speed of P. [4]

(b) A turntable has a rough horizontal surface, and it can rotate about a vertical axis through its centre O. While the turntable is stationary, a small object Q of mass 0.5 kg is placed on the turntable at a distance of 1.4 m from O. The turntable then begins to rotate, with a constant angular acceleration of 1.12 rad s^{-2}. Let ω rad s^{-1} be the angular speed of the turntable.

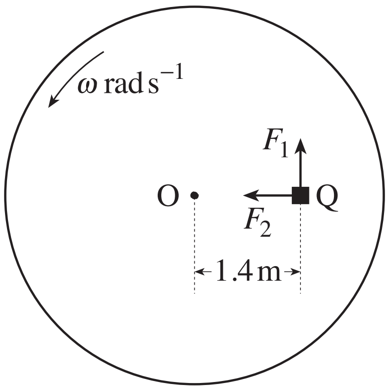

Fig. 2

(i) Given that Q does not slip, find the components F_1 and F_2 of the frictional force acting on Q perpendicular and parallel to QO (see Fig. 2). Give your answers in terms of ω where appropriate. [4]

The coefficient of friction between Q and the turntable is 0.65.

(ii) Find the value of ω when Q is about to slip. [5]

(iii) Find the angle which the frictional force makes with QO when Q is about to slip. [3]

[Turn over

3 A fixed point A is 12 m vertically above a fixed point B. A light elastic string, with natural length 3 m and modulus of elasticity 1323 N, has one end attached to A and the other end attached to a particle P of mass 15 kg. Another light elastic string, with natural length 4.5 m and modulus of elasticity 1323 N, has one end attached to B and the other end attached to P.

(i) Verify that, in the equilibrium position, AP = 5 m. [3]

The particle P now moves vertically, with both strings AP and BP remaining taut throughout the motion. The displacement of P above the equilibrium position is denoted by x m (see Fig. 3).

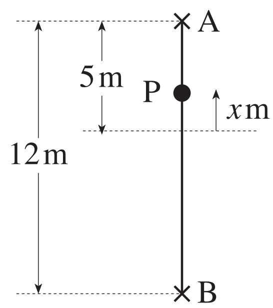

Fig. 3

(ii) Show that the tension in the string AP is $441(2-x)$ N and find the tension in the string BP. [3]

(iii) Show that the motion of P is simple harmonic, and state the period. [4]

The minimum length of AP during the motion is 3.5 m.

(iv) Find the maximum length of AP. [1]

(v) Find the speed of P when AP = 4.1 m. [3]

(vi) Find the time taken for AP to increase from 3.5 m to 4.5 m. [4]

4 The region bounded by the curve $y = \sqrt{x}$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(i) Find the x-coordinate of the centre of mass of this solid. [6]

From this solid, the cylinder with radius 1 and length 3 with its axis along the x-axis (from $x = 1$ to $x = 4$) is removed.

(ii) Show that the centre of mass of the remaining object, Q, has x-coordinate 3. [5]

This object Q has weight 96 N and it is supported, with its axis of symmetry horizontal, by a string passing through the cylindrical hole and attached to fixed points A and B (see Fig. 4). AB is horizontal and the sections of the string attached to A and B are vertical. There is sufficient friction to prevent slipping.

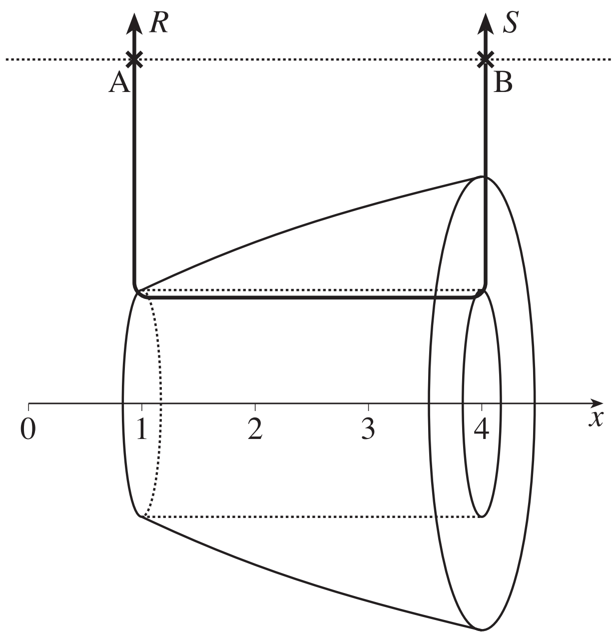

Fig. 4

(iii) Find the support forces, R and S, acting on the string at A and B

(A) when the string is light, [4]

(B) when the string is heavy and uniform with a total weight of 6 N. [3]

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4763/01**

MATHEMATICS (MEI)

Mechanics 3

WEDNESDAY 10 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

1 **(i)** Write down the dimensions of velocity, acceleration and force. [3]

The force F of gravitational attraction between two objects with masses m_1 and m_2, at a distance r apart, is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where G is the universal constant of gravitation.

(ii) Show that the dimensions of G are $\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}$. [2]

(iii) In SI units (based on the kilogram, metre and second) the value of G is 6.67×10^{-11}.

Find the value of G in imperial units based on the pound (0.4536 kg), foot (0.3048 m) and second. [3]

(iv) For a planet of mass m and radius r, the escape velocity v from the planet's surface is given by

$$v = \sqrt{\frac{2Gm}{r}}.$$

Show that this formula is dimensionally consistent. [3]

(v) For a planet in circular orbit of radius R round a star of mass M, the time t taken to complete one orbit is given by

$$t = kG^{\alpha}M^{\beta}R^{\gamma}$$

where k is a dimensionless constant.

Use dimensional analysis to find α, β and γ. [5]

2 **(a)** A light inextensible string has length 1.8 m. One end of the string is attached to a fixed point O, and the other end is attached to a particle of mass 5 kg. The particle moves in a complete vertical circle with centre O, so that the string remains taut throughout the motion. Air resistance may be neglected.

(i) Show that, at the highest point of the circle, the speed of the particle is at least $4.2\,\mathrm{m\,s^{-1}}$. [3]

(ii) Find the least possible tension in the string when the particle is at the lowest point of the circle. [5]

(b) Fig. 2 shows a hollow cone mounted with its axis of symmetry vertical and its vertex V pointing downwards. The cone rotates about its axis with a constant angular speed of $\omega\,\mathrm{rad\,s^{-1}}$. A particle P of mass 0.02 kg is in contact with the rough inside surface of the cone, and does not slip. The particle P moves in a horizontal circle of radius 0.32 m. The angle between VP and the vertical is θ.

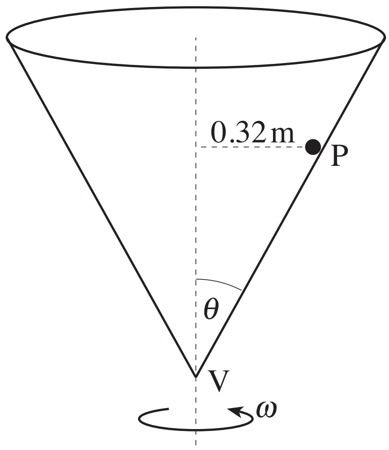

Fig. 2

In the case when $\omega = 8.75$, there is no frictional force acting on P.

(i) Show that $\tan\theta = 0.4$. [4]

Now consider the case when ω takes a constant value greater than 8.75.

(ii) Draw a diagram showing the forces acting on P. [2]

(iii) You are given that the coefficient of friction between P and the surface is 0.11. Find the maximum possible value of ω for which the particle does not slip. [6]

3 Ben has mass 60 kg and he is considering doing a bungee jump using an elastic rope with natural length 32 m. One end of the rope is attached to a fixed point O, and the other end is attached to Ben. When Ben is supported in equilibrium by the rope, the length of the rope is 32.8 m.

To predict what will happen, Ben is modelled as a particle B, the rope is assumed to be light, and air resistance is neglected. B is released from rest at O and falls vertically. When the rope becomes stretched, x m denotes the extension of the rope.

(i) Find the stiffness of the rope. [2]

(ii) Use an energy argument to show that, when B comes to rest instantaneously with the rope stretched,

$$x^2 - 1.6x - 51.2 = 0.$$

Hence find the length of the rope when B is at its lowest point. [6]

(iii) Show that, while the rope is stretched,

$$\frac{d^2x}{dt^2} + 12.25x = 9.8$$

where t is the time measured in seconds. [4]

(iv) Find the time taken for B to travel between the equilibrium position ($x = 0.8$) and the lowest point. [3]

(v) Find the acceleration of B when it is at the lowest point, and comment on the implications for Ben. [3]

4 In this question, a is a constant with $a > 1$.

Fig. 4 shows the region bounded by the curve $y = \frac{1}{x^2}$ for $1 \leqslant x \leqslant a$, the x-axis, and the lines $x = 1$ and $x = a$.

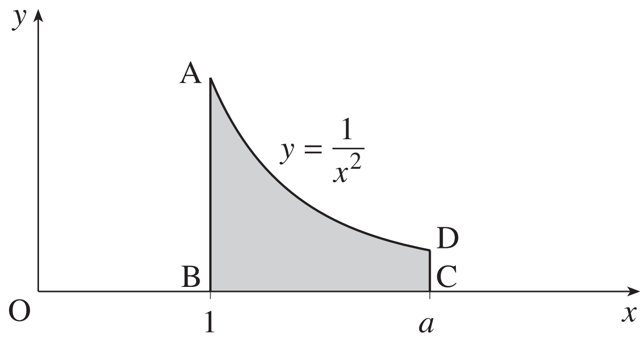

Fig. 4

This region is occupied by a uniform lamina ABCD, where A is $(1, 1)$, B is $(1, 0)$, C is $(a, 0)$ and D is $\left(a, \frac{1}{a^2}\right)$. The centre of mass of this lamina is $(\bar{x}, \bar{y})$.

(i) Find $\bar{x}$ in terms of a, and show that $\bar{y} = \frac{a^3 - 1}{6(a^3 - a^2)}$. [8]

(ii) In the case $a = 2$, the lamina is freely suspended from the point A, and hangs in equilibrium. Find the angle which AB makes with the vertical. [3]

The region shown in Fig. 4 is now rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(iii) Find the x-coordinate of the centre of mass of this solid of revolution, in terms of a, and show that it is less than 1.5. [7]

ADVANCED GCE UNIT

4763/01

MATHEMATICS (MEI)

Mechanics 3

MONDAY 21 MAY 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

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ADVICE TO CANDIDATES

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This document consists of **6** printed pages and **2** blank pages.

  [Turn over

1 **(a)** **(i)** Write down the dimensions of the following quantities.

Velocity
Acceleration
Force
Density (which is mass per unit volume)
Pressure (which is force per unit area) [5]

For a fluid with constant density ρ, the velocity v, pressure P and height h at points on a streamline are related by Bernoulli's equation

$$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant},$$

where g is the acceleration due to gravity.

(ii) Show that the left-hand side of Bernoulli's equation is dimensionally consistent. [4]

(b) In a wave tank, a float is performing simple harmonic motion with period 3.49 s in a vertical line. The height of the float above the bottom of the tank is h m at a time t s. When $t = 0$, the height has its maximum value. The value of h varies between 1.6 and 2.2.

(i) Sketch a graph showing how h varies with t. [2]

(ii) Express h in terms of t. [4]

(iii) Find the magnitude and direction of the acceleration of the float when $h = 1.7$. [3]

2 A fixed hollow sphere with centre O has an inside radius of 2.7 m. A particle P of mass 0.4 kg moves on the smooth inside surface of the sphere.

At first, P is moving in a horizontal circle with constant speed, and OP makes a constant angle of 60° with the vertical (see Fig. 2.1).

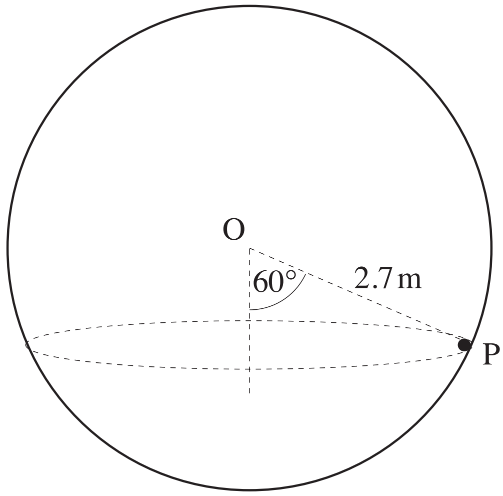

Fig. 2.1

(i) Find the normal reaction acting on P. [2]

(ii) Find the speed of P. [4]

The particle P is now placed at the lowest point of the sphere and is given an initial horizontal speed of 9 m s^{-1}. It then moves in part of a vertical circle. When OP makes an angle θ with the upward vertical and P is still in contact with the sphere, the speed of P is v m s^{-1} and the normal reaction acting on P is R N (see Fig. 2.2).

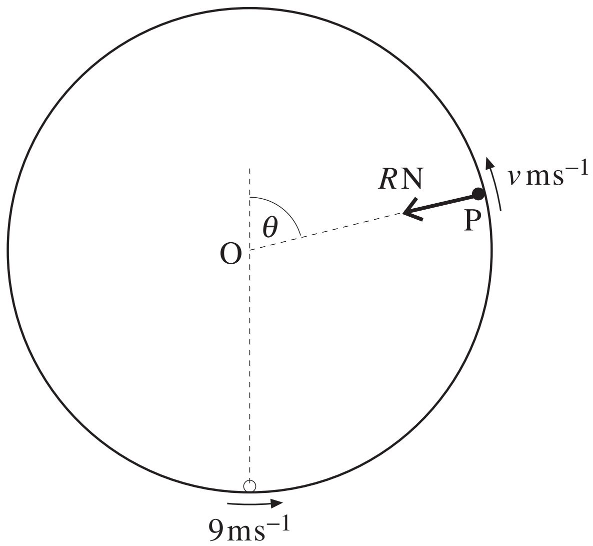

Fig. 2.2

(iii) Find v^2 in terms of θ. [3]

(iv) Show that $R = 4.16 - 11.76\cos\theta$. [5]

(v) Find the speed of P at the instant when it leaves the surface of the sphere. [4]

3 A light elastic string has natural length 1.2 m and stiffness 637 N m^{-1}.

(i) The string is stretched to a length of 1.3 m. Find the tension in the string and the elastic energy stored in the string. [3]

One end of this string is attached to a fixed point A. The other end is attached to a heavy ring R which is free to move along a smooth vertical wire. The shortest distance from A to the wire is 1.2 m (see Fig. 3).

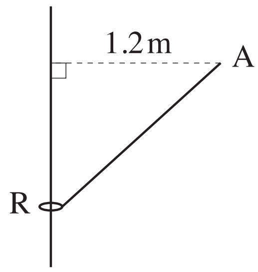

Fig. 3

The ring is in equilibrium when the length of the string AR is 1.3 m.

(ii) Show that the mass of the ring is 2.5 kg. [4]

The ring is given an initial speed u m s^{-1} vertically downwards from its equilibrium position. It first comes to rest, instantaneously, in the position where the length of AR is 1.5 m.

(iii) Find u. [7]

(iv) Determine whether the ring will rise above the level of A. [4]

4 **(a)** The region bounded by the curve $y = x^3$ for $0 \leqslant x \leqslant 2$, the x-axis and the line $x = 2$, is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [8]

(b) The region bounded by the circular arc $y = \sqrt{4 - x^2}$ for $1 \leqslant x \leqslant 2$, the x-axis and the line $x = 1$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution, as shown in Fig. 4.1.

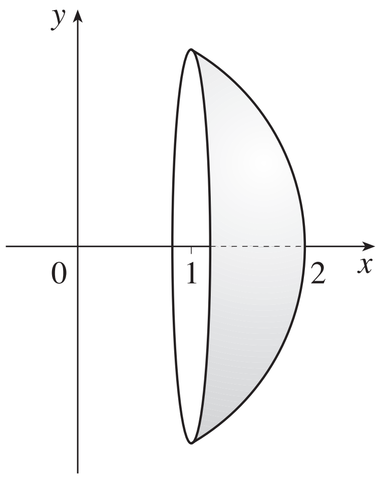

Fig. 4.1

(i) Show that the x-coordinate of the centre of mass of this solid of revolution is 1.35. [6]

This solid is placed on a rough horizontal surface, with its flat face in a vertical plane. It is held in equilibrium by a light horizontal string attached to its highest point and perpendicular to its flat face, as shown in Fig. 4.2.

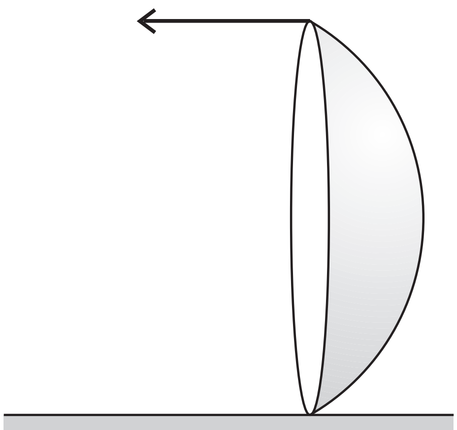

Fig. 4.2

(ii) Find the least possible coefficient of friction between the solid and the horizontal surface. [4]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4763/01**

MATHEMATICS (MEI)

Mechanics 3

THURSDAY 17 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
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INSTRUCTIONS TO CANDIDATES

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This document consists of **4** printed pages.

[Turn over

1 **(a)** **(i)** Write down the dimensions of force and the dimensions of density. [2]

When a wire, with natural length l_0 and cross-sectional area A, is stretched to a length l, the tension F in the wire is given by

$$F = \frac{EA(l - l_0)}{l_0}$$

where E is Young's modulus for the material from which the wire is made.

(ii) Find the dimensions of Young's modulus E. [3]

A uniform sphere of radius r is made from material with density ρ and Young's modulus E. When the sphere is struck, it vibrates with periodic time t given by

$$t = kr^{\alpha}\rho^{\beta}E^{\gamma}$$

where k is a dimensionless constant.

(iii) Use dimensional analysis to find α, β and γ. [5]

(b) Fig. 1 shows a fixed point A that is 1.5 m vertically above a point B on a rough horizontal surface. A particle P of mass 5 kg is at rest on the surface at a distance 0.8 m from B, and is connected to A by a light elastic string with natural length 1.5 m.

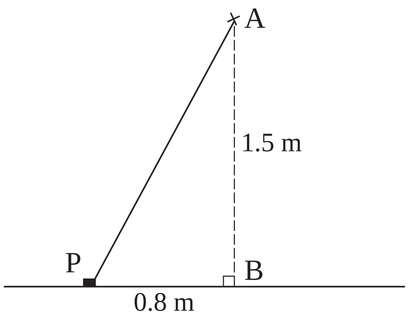

Fig. 1

The coefficient of friction between P and the surface is 0.4, and P is on the point of sliding. Find the stiffness of the string. [8]

2 **(a)** A small ball of mass 0.01 kg is moving in a vertical circle of radius 0.55 m on the smooth inside surface of a fixed sphere also of radius 0.55 m. When the ball is at the highest point of the circle, the normal reaction between the surface and the ball is 0.1 N. Modelling the ball as a particle and neglecting air resistance, find

(i) the speed of the ball when it is at the highest point of the circle, [3]

(ii) the normal reaction between the surface and the ball when the vertical height of the ball above the lowest point of the circle is 0.15 m. [5]

(b) A small object Q of mass 0.8 kg moves in a circular path, with centre O and radius r metres, on a smooth horizontal surface. A light elastic string, with natural length 2 m and modulus of elasticity 160 N, has one end attached to Q and the other end attached to O. The object Q has a constant angular speed of ω rad s^{-1}.

(i) Show that $\omega^2 = \dfrac{100(r-2)}{r}$ and deduce that $\omega < 10$. [4]

(ii) Find expressions, in terms of r only, for the elastic energy stored in the string, and for the kinetic energy of Q. Show that the kinetic energy of Q is greater than the elastic energy stored in the string. [4]

(iii) Given that the angular speed of Q is 6 rad s^{-1}, find the tension in the string. [3]

3 A particle is oscillating in a vertical line. At time t seconds, its displacement above the centre of the oscillations is x metres, where $x = A\sin\omega t + B\cos\omega t$ (and A, B and ω are constants).

(i) Show that $\dfrac{d^2x}{dt^2} = -\omega^2 x$. [3]

When $t = 0$, the particle is 2 m *above* the centre of the oscillations, the velocity is 1.44 m s^{-1} *downwards*, and the acceleration is 0.18 m s^{-2} *downwards*.

(ii) Find A, B and ω. [6]

(iii) Show that the period of oscillation is 20.9 s (correct to 3 significant figures), and find the amplitude. [3]

(iv) Find the total distance travelled by the particle between $t = 12$ and $t = 24$. [5]

[Question 4 is printed overleaf.]

4 Fig. 4.1 shows the region R bounded by the curve $y = x^{-\frac{1}{3}}$ for $1 \leqslant x \leqslant 8$, the x-axis, and the lines $x = 1$ and $x = 8$.

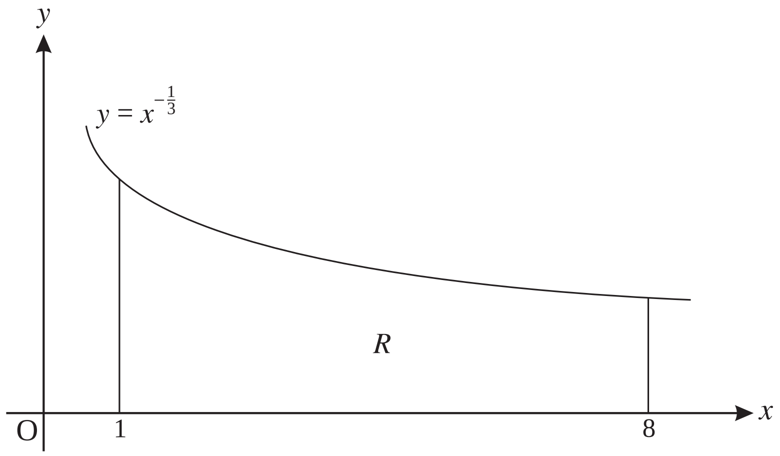

Fig. 4.1

(i) Find the x-coordinate of the centre of mass of a uniform solid of revolution obtained by rotating R through 2π radians about the x-axis. [6]

(ii) Find the coordinates of the centre of mass of a uniform lamina in the shape of the region R. [8]

(iii) Using your answer to part **(ii)**, or otherwise, find the coordinates of the centre of mass of a uniform lamina in the shape of the region (shown shaded in Fig. 4.2) bounded by the curve $y = x^{-\frac{1}{3}}$ for $1 \leqslant x \leqslant 8$, the line $y = \frac{1}{2}$ and the line $x = 1$. [4]

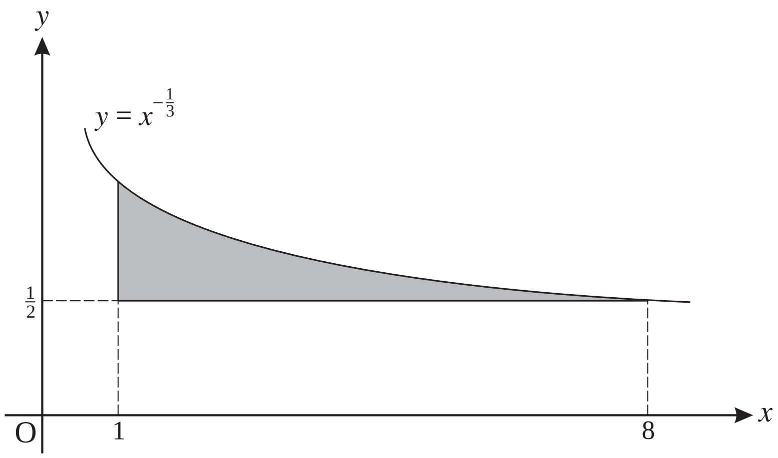

Fig. 4.2

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4763/01**

MATHEMATICS (MEI)

Mechanics 3

FRIDAY 23 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

1 **(a)** **(i)** Write down the dimensions of velocity, acceleration and force. [3]

A ball of mass m is thrown vertically upwards with initial velocity U. When the velocity of the ball is v, it experiences a force λv^2 due to air resistance where λ is a constant.

(ii) Find the dimensions of λ. [2]

A formula approximating the greatest height H reached by the ball is

$$H \approx \frac{U^2}{2g} - \frac{\lambda U^4}{4mg^2}$$

where g is the acceleration due to gravity.

(iii) Show that this formula is dimensionally consistent. [4]

A better approximation has the form $H \approx \dfrac{U^2}{2g} - \dfrac{\lambda U^4}{4mg^2} + \frac{1}{6}\lambda^2 U^{\alpha} m^{\beta} g^{\gamma}$.

(iv) Use dimensional analysis to find α, β and γ. [5]

(b) A girl of mass 50 kg is practising for a bungee jump. She is connected to a fixed point O by a light elastic rope with natural length 24 m and modulus of elasticity 2060 N. At one instant she is 30 m vertically below O and is moving vertically upwards with speed 12 m s^{-1}. She comes to rest instantaneously, with the rope slack, at the point A. Find the distance OA. [4]

2 A particle P of mass 0.3 kg is connected to a fixed point O by a light inextensible string of length 4.2 m.

Firstly, P is moving in a horizontal circle as a conical pendulum, with the string making a constant angle with the vertical. The tension in the string is 3.92 N.

(i) Find the angle which the string makes with the vertical. [2]

(ii) Find the speed of P. [4]

P now moves in part of a vertical circle with centre O and radius 4.2 m. When the string makes an angle θ with the downward vertical, the speed of P is v m s^{-1} (see Fig. 2). You are given that $v = 8.4$ when $\theta = 60°$.

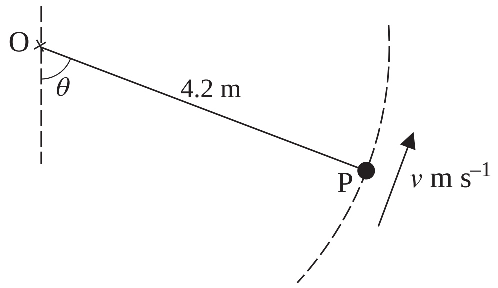

Fig. 2

(iii) Find the tension in the string when $\theta = 60°$. [3]

(iv) Show that $v^2 = 29.4 + 82.32\cos\theta$. [4]

(v) Find θ at the instant when the string becomes slack. [5]

3 A small block B has mass 2.5 kg. A light elastic string connects B to a fixed point P, and a second light elastic string connects B to a fixed point Q, which is 6.5 m vertically below P.

The string PB has natural length 3.2 m and stiffness $35\,\mathrm{N\,m^{-1}}$; the string BQ has natural length 1.8 m and stiffness $5\,\mathrm{N\,m^{-1}}$.

The block B is released from rest in the position 4.4 m vertically below P. You are given that B performs simple harmonic motion along part of the line PQ, and that both strings remain taut throughout the motion. Air resistance may be neglected. At time t seconds after release, the length of the string PB is x metres (see Fig. 3).

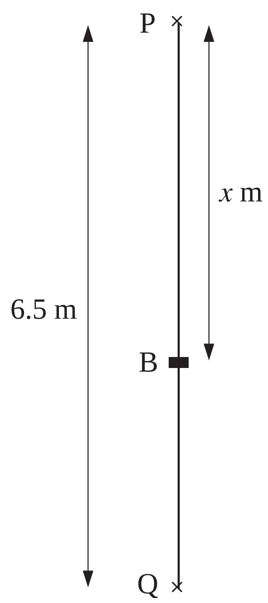

Fig. 3

(i) Find, in terms of x, the tension in the string PB and the tension in the string BQ. [3]

(ii) Show that $\dfrac{d^2x}{dt^2} = 64 - 16x$. [4]

(iii) Find the value of x when B is at the centre of oscillation. [2]

(iv) Find the period of oscillation. [2]

(v) Write down the amplitude of the motion and find the maximum speed of B. [3]

(vi) Find the time after release when B is first moving *downwards* with speed $0.9\,\mathrm{m\,s^{-1}}$. [4]

[Question 4 is printed overleaf.]

4 **(a)** A uniform solid of revolution is obtained by rotating through 2π radians about the y-axis the region bounded by the curve $y = 8 - 2x^2$ for $0 \leqslant x \leqslant 2$, the x-axis and the y-axis.

(i) Find the y-coordinate of the centre of mass of this solid. [7]

The solid is now placed on a rough plane inclined at an angle θ to the horizontal. It rests in equilibrium with its circular face in contact with the plane as shown in Fig. 4.

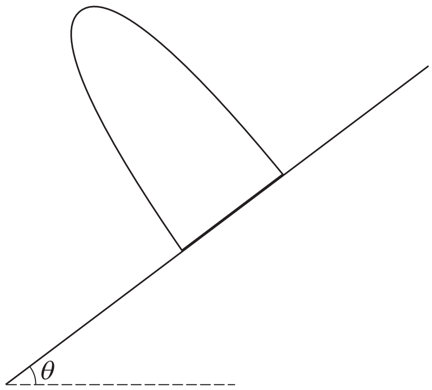

Fig. 4

(ii) Given that the solid is on the point of toppling, find θ. [4]

(b) Find the y-coordinate of the centre of mass of a uniform lamina in the shape of the region bounded by the curve $y = 8 - 2x^2$ for $-2 \leqslant x \leqslant 2$, and the x-axis. [7]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 21 January 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(i)** Write down the dimensions of force and density (which is mass per unit volume). **[2]**

The viscosity, η, of a fluid is defined by the equation

$$F = \frac{\eta A(v_2 - v_1)}{d}$$

where F is the force acting over an area A, and v_1, v_2 are the velocities at two points a distance d apart in the fluid.

(ii) Find the dimensions of viscosity. **[3]**

(iii) When a sphere of radius a and density ρ falls through a fluid with viscosity η, it reaches a terminal velocity v given by $v = \frac{2a^2\rho g}{9\eta}$. Show that this formula is dimensionally consistent. **[3]**

The Reynolds number, R, for the flow of fluid round an obstruction of width w is a dimensionless quantity given by

$$R = \rho w^{\alpha} v^{\beta} \eta^{\gamma}$$

where v is the velocity of the flow, ρ is the density of the fluid and η its viscosity.

(iv) Find the values of α, β and γ. **[5]**

A designer is investigating the flow of air round an aircraft of width 25 moving with velocity 150, at a height where the air has density 0.4 and viscosity 1.6×10^{-5} (all in SI units). A scale model of the aircraft, with width 5, is used in a wind tunnel at ground level, where the air has density 1.3 and viscosity 1.8×10^{-5}. The Reynolds number for the model must be the same as that for the full-sized aircraft.

(v) Find the velocity of flow required in the wind tunnel. **[3]**

2 **(a)** Fig. 2 shows a light inextensible string of length 3.3 m passing through a small **smooth** ring R of mass 0.27 kg. The ends of the string are attached to fixed points A and B, where A is vertically above B. The ring R is moving with constant speed in a horizontal circle of radius 1.2 m, AR = 2.0 m and BR = 1.3 m.

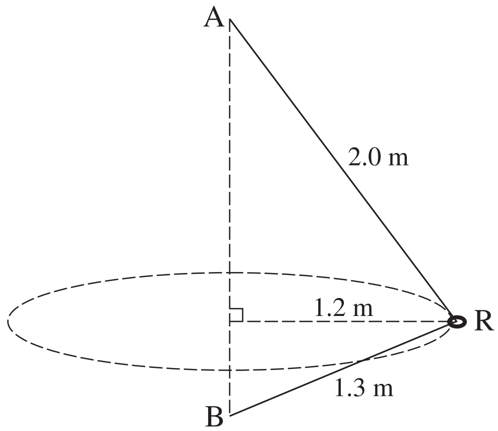

Fig. 2

(i) Show that the tension in the string is 6.37 N. **[5]**

(ii) Find the speed of R. **[4]**

(b) One end of a light inextensible string of length 1.25 m is attached to a fixed point O. The other end is attached to a particle P of mass 0.2 kg. The particle P is moving in a vertical circle with centre O and radius 1.25 m, and when P is at the highest point of the circle there is no tension in the string.

(i) Show that when P is at the highest point its speed is $3.5\ \mathrm{m\,s^{-1}}$. **[2]**

For the instant when the string OP makes an angle of $60°$ with the upward vertical, find

(ii) the radial and tangential components of the acceleration of P, **[6]**

(iii) the tension in the string. **[2]**

3 An elastic rope has natural length 25 m and modulus of elasticity 980 N. One end of the rope is attached to a fixed point O, and a rock of mass 5 kg is attached to the other end; the rock is always vertically below O.

(i) Find the extension of the rope when the rock is hanging in equilibrium. **[2]**

When the rock is moving with the rope stretched, its displacement is x metres below the equilibrium position at time t seconds.

(ii) Show that $\frac{d^2x}{dt^2} = -7.84x$. **[4]**

The rock is released from a position where the rope is slack, and when the rope just becomes taut the speed of the rock is $8.4\,\mathrm{m\,s^{-1}}$.

(iii) Find the distance below the equilibrium position at which the rock first comes instantaneously to rest. **[4]**

(iv) Find the maximum speed of the rock. **[2]**

(v) Find the time between the rope becoming taut and the rock first coming to rest. **[4]**

(vi) State three modelling assumptions you have made in answering this question. **[3]**

4 **(a)** The region bounded by the x-axis and the semicircle $y = \sqrt{a^2 - x^2}$ for $-a \leqslant x \leqslant a$ is occupied by a uniform lamina with area $\frac{1}{2}\pi a^2$. Show by integration that the y-coordinate of the centre of mass of this lamina is $\dfrac{4a}{3\pi}$. **[4]**

(b) A uniform solid cone is formed by rotating the region between the x-axis and the line $y = mx$, for $0 \leqslant x \leqslant h$, through 2π radians about the x-axis.

(i) Show that the x-coordinate of the centre of mass of this cone is $\frac{3}{4}h$. **[6]**
[You may use the formula $\frac{1}{3}\pi r^2 h$ for the volume of a cone.]

From such a uniform solid cone with radius 0.7 m and height 2.4 m, a cone of material is removed. The cone removed has radius 0.4 m and height 1.1 m; the centre of its base coincides with the centre of the base of the original cone, and its axis of symmetry is also the axis of symmetry of the original cone. Fig. 4 shows the resulting object; the vertex of the original cone is V, and A is a point on the circumference of its base.

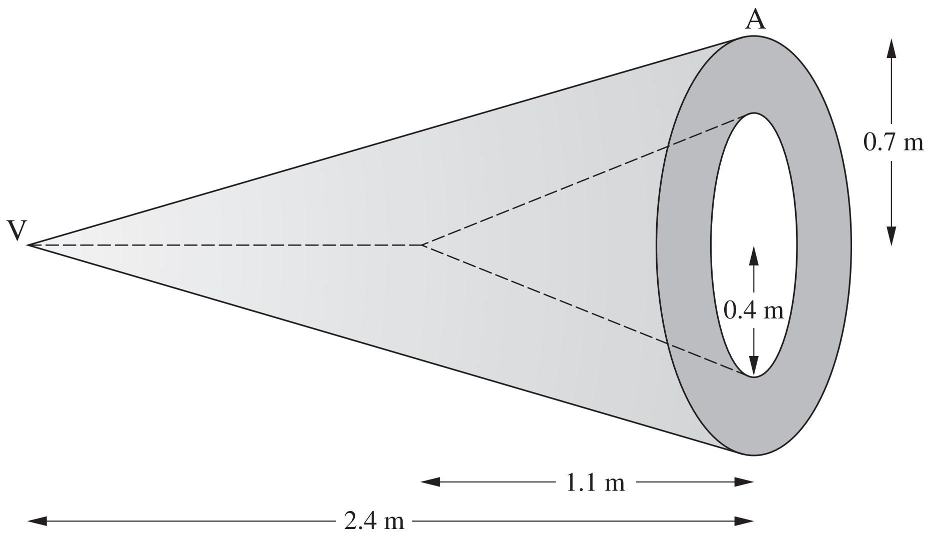

Fig. 4

(ii) Find the distance of the centre of mass of this object from V. **[5]**

This object is suspended by a string attached to a point Q on the line VA, and hangs in equilibrium with VA horizontal.

(iii) Find the distance VQ. **[3]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 19 June 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
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INFORMATION FOR CANDIDATES

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- This document consists of **4** pages. Any blank pages are indicated.

1 A fixed solid sphere has centre O and radius 2.6 m. A particle P of mass 0.65 kg moves on the smooth surface of the sphere.

The particle P is set in motion with horizontal velocity $1.4\,\text{m}\,\text{s}^{-1}$ at the highest point of the sphere, and moves in part of a vertical circle. When OP makes an angle θ with the upward vertical, and P is still in contact with the sphere, the speed of P is $v\,\text{m}\,\text{s}^{-1}$.

(i) Show that $v^2 = 52.92 - 50.96\cos\theta$. **[3]**

(ii) Find, in terms of θ, the normal reaction acting on P. **[4]**

(iii) Find the speed of P at the instant when it leaves the surface of the sphere. **[4]**

The particle P is now attached to one end of a light inextensible string, and the other end of the string is fixed to a point A, vertically above O, such that AP is tangential to the sphere, as shown in Fig. 1. P moves with constant speed $1.2\,\text{m}\,\text{s}^{-1}$ in a **horizontal** circle with radius 2.4 m on the surface of the sphere.

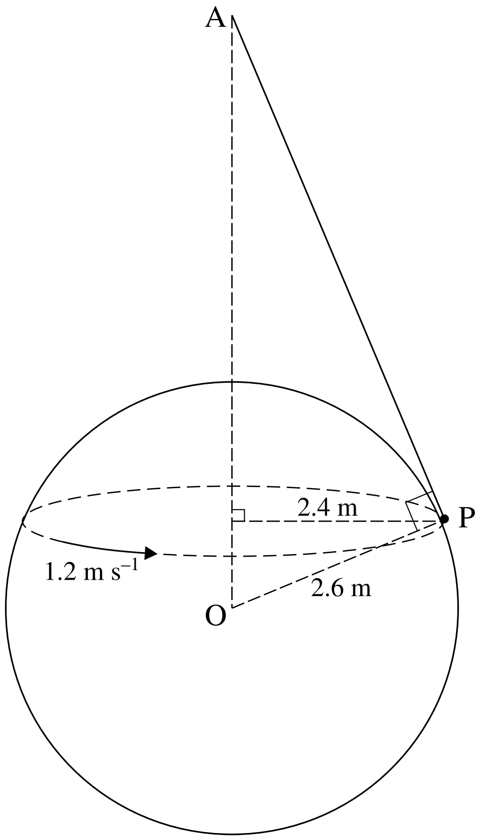

Fig. 1

(iv) Find the tension in the string and the normal reaction acting on P. **[8]**

2 In trials for a vehicle emergency stopping system, a small car of mass 400 kg is propelled towards a buffer. The buffer is modelled as a light spring of stiffness $5000\,\mathrm{N\,m^{-1}}$. One end of the spring is fixed, and the other end points directly towards the oncoming car. Throughout this question, there is no driving force acting on the car, and there are no resistances to motion apart from those specifically mentioned.

At first, the buffer is mounted on a horizontal surface, and the car has speed $3\,\mathrm{m\,s^{-1}}$ when it hits the buffer, as shown in Fig. 2.1.

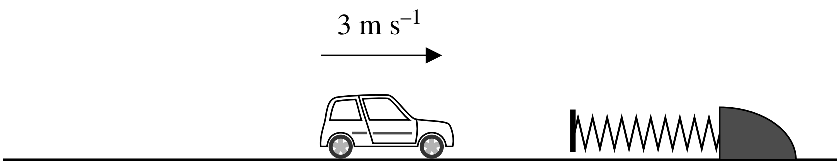

Fig. 2.1

(i) Find the compression of the spring when the car comes (instantaneously) to rest. **[3]**

The buffer is now mounted on a slope making an angle θ with the horizontal, where $\sin\theta = \frac{1}{7}$. The car is released from rest and travels 7.35 m down the slope before hitting the buffer, as shown in Fig. 2.2.

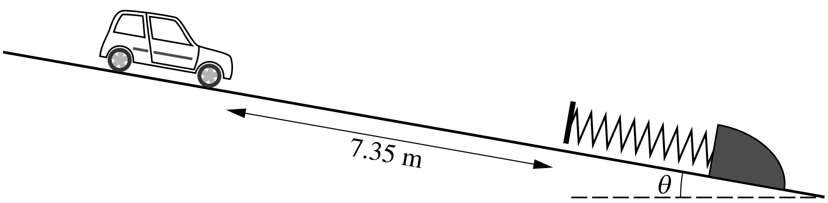

Fig. 2.2

(ii) Verify that the car comes instantaneously to rest when the spring is compressed by 1.4 m. **[4]**

The surface of the slope (including the section under the buffer) is now covered with gravel which exerts a constant resistive force of 7560 N on the car. The car is moving down the slope, and has speed $30\,\mathrm{m\,s^{-1}}$ when it is 24 m from the buffer, as shown in Fig. 2.3. It comes to rest when the spring has been compressed by x metres.

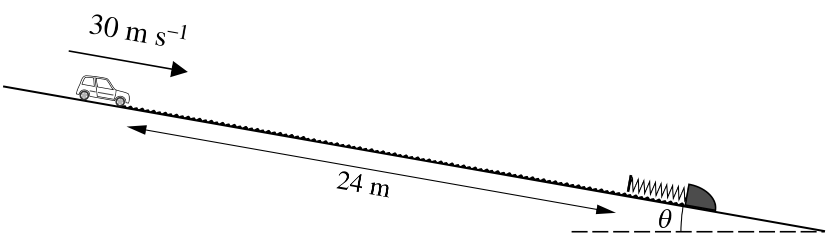

Fig. 2.3

(iii) By considering work and energy, find the value of x. **[10]**

3 **(a)** **(i)** Write down the dimensions of velocity, force and density (which is mass per unit volume). **[3]**

A vehicle moving with velocity v experiences a force F, due to air resistance, given by

$$F = \tfrac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma}$$

where ρ is the density of the air, A is the cross-sectional area of the vehicle, and C is a dimensionless quantity called the drag coefficient.

(ii) Use dimensional analysis to find α, β and γ. **[5]**

(b) A light rod is freely pivoted about a fixed point at one end and has a heavy weight attached to its other end. The rod with the weight attached is oscillating in a vertical plane as a simple pendulum with period 4.3 s. The maximum angle which the rod makes with the vertical is 0.08 radians. You may assume that the motion is simple harmonic.

(i) Find the angular speed of the rod when it makes an angle of 0.05 radians with the vertical. **[5]**

(ii) Find the time taken for the pendulum to swing directly from a position where the rod makes an angle of 0.05 radians on one side of the vertical to the position where the rod makes an angle of 0.05 radians on the other side of the vertical. **[5]**

4 **(a)** A uniform lamina occupies the region bounded by the x-axis, the y-axis, the curve $y = \mathrm{e}^x$ for $0 \leqslant x \leqslant \ln 3$, and the line $x = \ln 3$. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

(b) A region is bounded by the x-axis, the curve $y = \dfrac{6}{x^2}$ for $2 \leqslant x \leqslant a$ (where $a > 2$), the line $x = 2$ and the line $x = a$. This region is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(i) Show that the x-coordinate of the centre of mass of this solid is $\dfrac{3(a^3 - 4a)}{a^3 - 8}$. **[6]**

(ii) Show that, however large the value of a, the centre of mass of this solid is less than 3 units from the origin. **[3]**

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(a)** **(i)** Write down the dimensions of density, kinetic energy and power. **[3]**

A sphere of radius r is moved at constant velocity v through a fluid.

(ii) In a viscous fluid, the power required is $6\pi\eta r v^2$, where η is the viscosity of the fluid.

Find the dimensions of viscosity. **[3]**

(iii) In a non-viscous fluid, the power required is $k\rho^\alpha r^\beta v^\gamma$, where ρ is the density of the fluid and k is a dimensionless constant.

Use dimensional analysis to find α, β and γ. **[6]**

(b) A rock of mass 5.5 kg is connected to a fixed point O by a light elastic rope with natural length 1.2 m. The rock is released from rest in a position 2 m vertically below O, and it next comes to instantaneous rest when it is 1.5 m vertically above O.

Find the stiffness of the rope. **[6]**

2 **(a)** A uniform solid hemisphere of volume $\frac{2}{3}\pi a^3$ is formed by rotating the region bounded by the x-axis, the y-axis and the curve $y = \sqrt{a^2 - x^2}$ for $0 \leqslant x \leqslant a$, through 2π radians about the x-axis.

Show that the x-coordinate of the centre of mass of the hemisphere is $\frac{3}{8}a$. **[5]**

(b) A uniform lamina is bounded by the x-axis, the line $x = 1$, and the curve $y = 2 - \sqrt{x}$ for $1 \leqslant x \leqslant 4$. Its corners are A (1, 1), B (1, 0) and C (4, 0).

(i) Find the coordinates of the centre of mass of the lamina. **[9]**

The lamina is suspended with AB vertical and BC horizontal by light vertical strings attached to A and C, as shown in Fig. 2. The weight of the lamina is W.

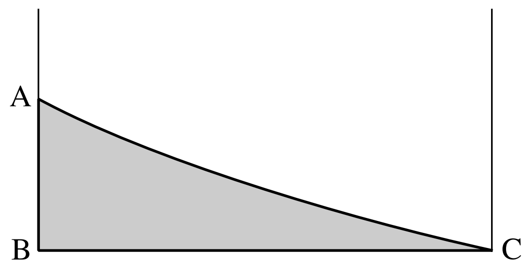

Fig. 2

(ii) Find the tensions in the two strings in terms of W. **[4]**

3 A particle P of mass 0.6 kg is connected to a fixed point O by a light inextensible string of length 1.25 m. When it is 1.25 m vertically below O, P is set in motion with horizontal velocity $6\,\mathrm{m\,s^{-1}}$ and then moves in part of a vertical circle with centre O and radius 1.25 m. When OP makes an angle θ with the downward vertical, the speed of P is $v\,\mathrm{m\,s^{-1}}$, as shown in Fig. 3.1.

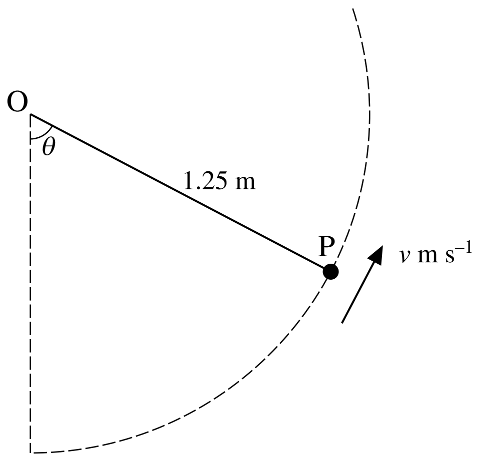

Fig. 3.1

(i) Show that $v^2 = 11.5 + 24.5\cos\theta$. **[3]**

(ii) Find the tension in the string in terms of θ. **[4]**

(iii) Find the speed of P at the instant when the string becomes slack. **[4]**

A second light inextensible string, of length 0.35 m, is attached to P, and the other end of this string is attached to a point C which is 1.2 m vertically below O. The particle P now moves in a horizontal circle with centre C and radius 0.35 m, as shown in Fig. 3.2. The speed of P is $1.4\,\mathrm{m\,s^{-1}}$.

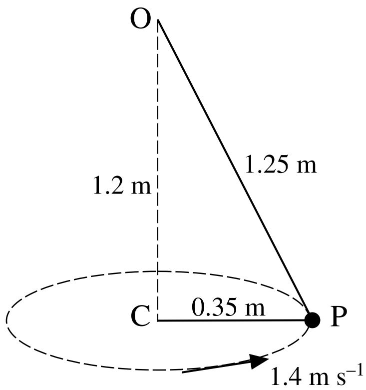

Fig. 3.2

(iv) Find the tension in the string OP and the tension in the string CP. **[7]**

[Question 4 is printed overleaf.]

4 Fig. 4 shows a smooth plane inclined at an angle of 30° to the horizontal. Two fixed points A and B on the plane are 4.55 m apart with B higher than A on a line of greatest slope. A particle P of mass 0.25 kg is in contact with the plane and is connected to A and to B by two light elastic strings. The string AP has natural length 1.5 m and modulus of elasticity 7.35 N; the string BP has natural length 2.5 m and modulus of elasticity 7.35 N. The particle P moves along part of the line AB, with both strings taut throughout the motion.

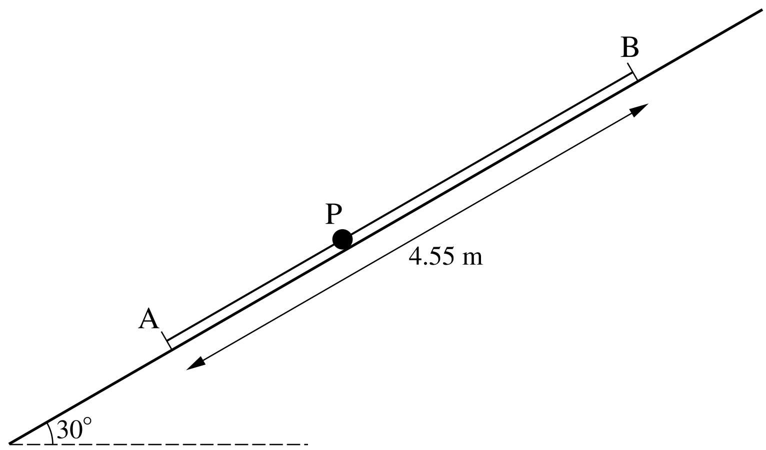

Fig. 4

(i) Show that, when AP = 1.55 m, the acceleration of P is zero. **[5]**

(ii) Taking AP = $(1.55 + x)$ m, write down the tension in the string AP, in terms of x, and show that the tension in the string BP is $(1.47 - 2.94x)$ N. **[3]**

(iii) Show that the motion of P is simple harmonic, and find its period. **[5]**

The particle P is released from rest with AP = 1.5 m.

(iv) Find the time after release when P is first moving *down* the plane with speed $0.2\,\mathrm{m\,s^{-1}}$. **[5]**

RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

4763

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Turn over

1 **(a)** Two light elastic strings, each having natural length 2.15 m and stiffness $70\,\mathrm{N\,m^{-1}}$, are attached to a particle P of mass 4.8 kg. The other ends of the strings are attached to fixed points A and B, which are 1.4 m apart at the same horizontal level. The particle P is placed 2.4 m vertically below the midpoint of AB, as shown in Fig. 1.

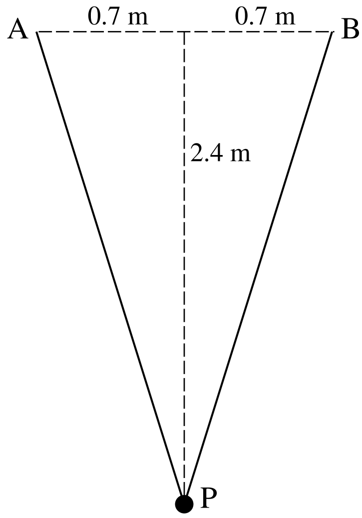

Fig. 1

(i) Show that P is in equilibrium in this position. **[6]**

(ii) Find the energy stored in the string AP. **[2]**

Starting in this equilibrium position, P is set in motion with initial velocity $3.5\,\mathrm{m\,s^{-1}}$ vertically upwards. You are given that P first comes to instantaneous rest at a point C where the strings are slack.

(iii) Find the vertical height of C above the initial position of P. **[4]**

(b) **(i)** Write down the dimensions of force and stiffness (of a spring). **[2]**

A particle of mass m is performing oscillations with amplitude a on the end of a spring with stiffness k. The maximum speed v of the particle is given by $v = cm^{\alpha}k^{\beta}a^{\gamma}$, where c is a dimensionless constant.

(ii) Use dimensional analysis to find α, β and γ. **[4]**

2 A hollow hemisphere has internal radius 2.5 m and is fixed with its rim horizontal and uppermost. The centre of the hemisphere is O. A small ball B of mass 0.4 kg moves in contact with the smooth inside surface of the hemisphere.

At first, B is moving at constant speed in a horizontal circle with radius 1.5 m, as shown in Fig. 2.1.

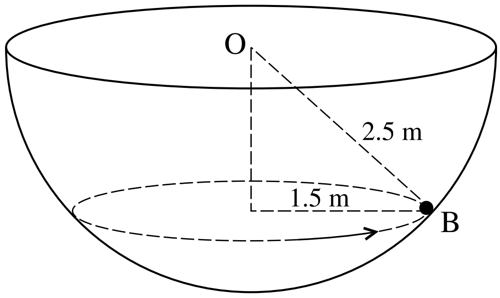

Fig. 2.1

(i) Find the normal reaction of the hemisphere on B. **[3]**

(ii) Find the speed of B. **[3]**

The ball B is now released from rest on the inside surface at a point on the same horizontal level as O. It then moves in part of a vertical circle with centre O and radius 2.5 m, as shown in Fig. 2.2.

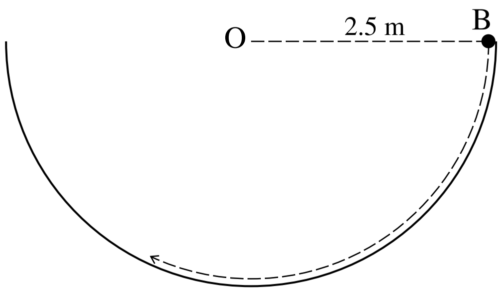

Fig. 2.2

(iii) Show that, when B is at its lowest point, the normal reaction is three times the weight of B. **[4]**

For an instant when the normal reaction is twice the weight of B, find

(iv) the speed of B, **[5]**

(v) the tangential component of the acceleration of B. **[3]**

3 In this question, give your answers in an exact form.

The region R_1 (shown in Fig. 3) is bounded by the x-axis, the lines $x = 1$ and $x = 5$, and the curve $y = \dfrac{1}{x}$ for $1 \leqslant x \leqslant 5$.

(i) A uniform solid of revolution is formed by rotating the region R_1 through 2π radians about the x-axis. Find the x-coordinate of the centre of mass of this solid. **[5]**

(ii) Find the coordinates of the centre of mass of a uniform lamina occupying the region R_1. **[7]**

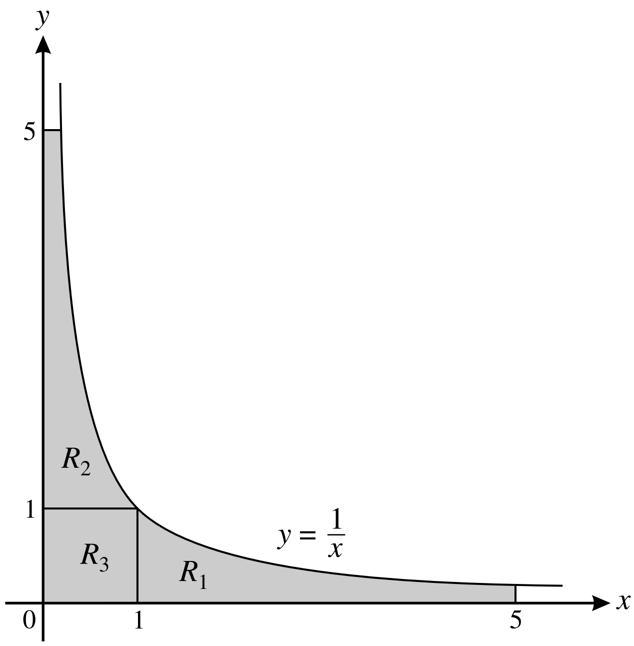

Fig. 3

The region R_2 is bounded by the y-axis, the lines $y = 1$ and $y = 5$, and the curve $y = \dfrac{1}{x}$ for $\frac{1}{5} \leqslant x \leqslant 1$.
The region R_3 is the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1).

(iii) Write down the coordinates of the centre of mass of a uniform lamina occupying the region R_2. **[2]**

(iv) Find the coordinates of the centre of mass of a uniform lamina occupying the region consisting of R_1, R_2 and R_3 (shown shaded in Fig. 3). **[4]**

4 A particle P is performing simple harmonic motion in a vertical line. At time t s, its displacement x m above a fixed point O is given by

$$x = A \sin \omega t + B \cos \omega t$$

where A, B and ω are constants.

(i) Show that the acceleration of P, in $\mathrm{m\,s^{-2}}$, is $-\omega^2 x$. **[3]**

When $t = 0$, P is 16 m *below* O, moving with velocity $7.5\ \mathrm{m\,s^{-1}}$ *upwards*, and has acceleration $1\ \mathrm{m\,s^{-2}}$ *upwards*.

(ii) Find the values of A, B and ω. **[4]**

(iii) Find the maximum displacement, the maximum speed, and the maximum acceleration of P. **[5]**

(iv) Find the speed and the direction of motion of P when $t = 15$. **[2]**

(v) Find the distance travelled by P between $t = 0$ and $t = 15$. **[4]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 26 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Turn over

1 **(i)** Write down the dimensions of force, density and angular speed. **[3]**

The breaking stress, S, of a material is defined by

$$S = \frac{F}{A}$$

where F is the force required to break a specimen with cross-sectional area A.

(ii) Show that the dimensions of breaking stress are $\mathrm{ML^{-1}T^{-2}}$. **[2]**

In SI units (based on kilograms, metres and seconds), the unit of breaking stress is the pascal (Pa). The breaking stress of steel is 1.2×10^9 Pa.

(iii) Find the breaking stress of steel when expressed in a new system of units based on pounds, inches and milliseconds, where 1 pound = 0.454 kg, 1 inch = 0.0254 m and 1 millisecond = 0.001 s. **[3]**

A material has breaking stress S and density ρ. When a disc of radius r, made from this material, is rotated very quickly, there is a critical angular speed at which the disc will break apart. This critical angular speed, ω, is given by

$$\omega = kS^{\alpha}\rho^{\beta}r^{\gamma}$$

where k is a dimensionless constant.

(iv) Use dimensional analysis to find α, β and γ. **[4]**

Steel has breaking stress 1.2×10^9 Pa and density $7800\,\mathrm{kg\,m^{-3}}$. For a steel disc of radius 0.5 m the critical angular speed is $3140\,\mathrm{rad\,s^{-1}}$. Aluminium has density $2700\,\mathrm{kg\,m^{-3}}$ and for an aluminium disc of radius 0.2 m the critical angular speed is $8120\,\mathrm{rad\,s^{-1}}$.

(v) Find the breaking stress of aluminium. **[4]**

Using a different system of units, a disc of radius 15 is made from material with breaking stress 630 and density 70.

(vi) Find, in these units, the critical angular speed for this disc. **[2]**

2 **(a)** A particle P, of mass 48 kg, is moving in a horizontal circle of radius 8.4 m at a constant speed of V m s^{-1}, in contact with a smooth horizontal surface. A light inextensible rope of length 30 m connects P to a fixed point A which is vertically above the centre C of the circle, as shown in Fig. 2.1.

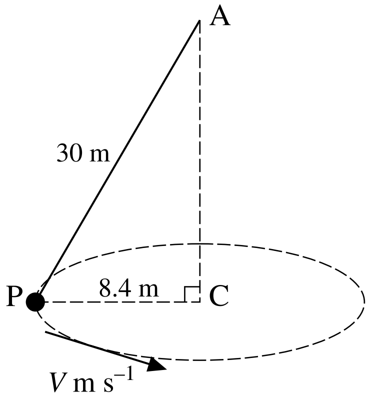

Fig. 2.1

(i) Given that $V = 3.5$, find the tension in the rope and the normal reaction of the surface on P. **[5]**

(ii) Calculate the value of V for which the normal reaction is zero. **[4]**

(b) The particle P, of mass 48 kg, is now placed on the highest point of a fixed solid sphere with centre O and radius 2.5 m. The surface of the sphere is smooth. The particle P is given an initial horizontal velocity of u m s^{-1}, and it then moves in part of a vertical circle with centre O and radius 2.5 m. When OP makes an angle θ with the upward vertical and P is still in contact with the surface of the sphere, P has speed v m s^{-1} and the normal reaction of the sphere on P is R N, as shown in Fig. 2.2.

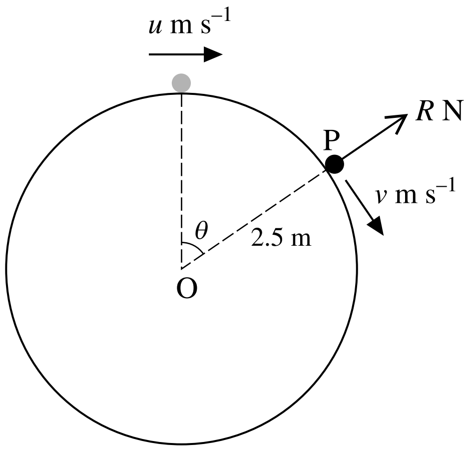

Fig. 2.2

(i) Show that $v^2 = u^2 + 49 - 49\cos\theta$. **[3]**

(ii) Find an expression for R in terms of u and v. **[4]**

(iii) Given that P loses contact with the surface of the sphere at the instant when its speed is 4.15 m s^{-1}, find the value of u. **[2]**

3 A block of mass 200 kg is connected to a horizontal ceiling by four identical light elastic ropes, each having natural length 7 m and stiffness $180\,\mathrm{N\,m^{-1}}$. It is also connected to the floor by a single light elastic rope having stiffness $80\,\mathrm{N\,m^{-1}}$. Throughout this question you may assume that all five ropes are stretched and vertical, and you may neglect air resistance.

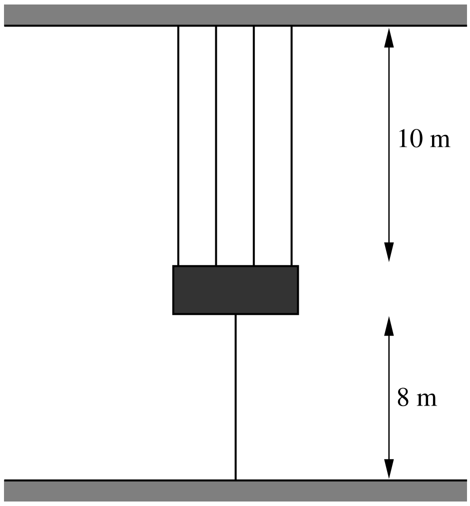

Fig. 3

Fig. 3 shows the block resting in equilibrium, with each of the top ropes having length 10 m and the bottom rope having length 8 m.

(i) Find the tension in one of the top ropes. **[2]**

(ii) Find the natural length of the bottom rope. **[4]**

The block now moves vertically up and down. At time t seconds, the block is x metres below its equilibrium position.

(iii) Show that $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4x$. **[4]**

The motion is started by pulling the block down 2.2 m below its equilibrium position and releasing it from rest. The block then executes simple harmonic motion with amplitude 2.2 m.

(iv) Find the maximum magnitude of the acceleration of the block. **[2]**

(v) Find the speed of the block when it has travelled 3.8 m from its starting point. **[2]**

(vi) Find the distance travelled by the block in the first 5 s. **[4]**

4 **(a)**

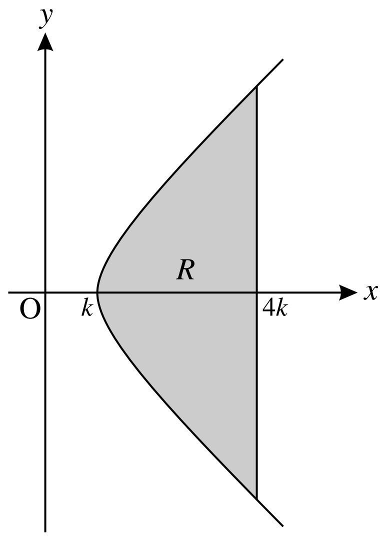

Fig. 4.1

The region R, shown in Fig. 4.1, is bounded by the curve $x^2 - y^2 = k^2$ for $k \leqslant x \leqslant 4k$ and the line $x = 4k$, where k is a positive constant. Find the x-coordinate of the centre of mass of the uniform solid of revolution formed when R is rotated about the x-axis. **[7]**

(b) A uniform lamina occupies the region bounded by the curve $y = \dfrac{x^3}{a^2}$ for $0 \leqslant x \leqslant 2a$, the x-axis and the line $x = 2a$, where a is a positive constant. The vertices of the lamina are O $(0, 0)$, A $(2a, 8a)$ and B $(2a, 0)$, as shown in Fig. 4.2.

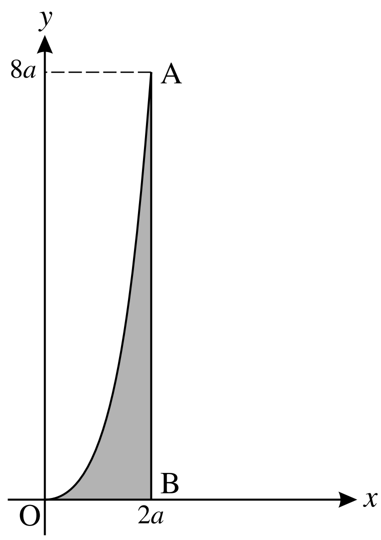

Fig. 4.2

(i) Find the coordinates of the centre of mass of the lamina. **[8]**

(ii) The lamina is freely suspended from the point A and hangs in equilibrium. Find the angle that AB makes with the vertical. **[3]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4763**

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 22 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Turn over

1 A particle is moving in a straight line. At time t its displacement x from a fixed point O on the line is given by

$$x = A \sin \omega t$$

where A and ω are constants.

(i) Show that $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$ and $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = \omega^2(A^2 - x^2)$. **[5]**

A ball floats on the surface of the sea. Waves cause the ball to rise and fall in a vertical line, and the ball is executing simple harmonic motion. The centre of the oscillations is 8 m above the sea-bed. The ball has speed $1.2\,\mathrm{m\,s^{-1}}$ when it is 7.3 m above the sea-bed, and it has speed $0.75\,\mathrm{m\,s^{-1}}$ when it is 10 m above the sea-bed.

(ii) Show that the amplitude of the oscillations is 2.5 m, and find the period. **[6]**

(iii) Find the maximum speed of the ball. **[1]**

(iv) Find the magnitude and direction of the acceleration of the ball when it is 6.4 m above the sea-bed. **[3]**

(v) Find the time taken for the ball to move upwards from 6 m above the sea-bed to 9 m above the sea-bed. **[3]**

2 **(a)** A particle P of mass 0.2 kg is connected to a fixed point O by a light inextensible string of length 3.2 m, and is moving in a vertical circle with centre O and radius 3.2 m. Air resistance may be neglected. When P is at the highest point of the circle, the tension in the string is 0.6 N.

(i) Find the speed of P when it is at the highest point. **[3]**

(ii) For an instant when OP makes an angle of 60° with the downward vertical, find

(A) the radial and tangential components of the acceleration of P, **[5]**

(B) the tension in the string. **[3]**

(b) A solid cone is fixed with its axis of symmetry vertical and its vertex V uppermost. The semi-vertical angle of the cone is 36°, and its surface is smooth. A particle Q of mass 0.2 kg is connected to V by a light inextensible string, and Q moves in a horizontal circle at constant speed, in contact with the surface of the cone, as shown in Fig. 2.

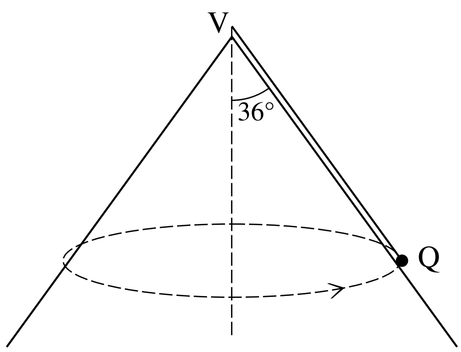

Fig. 2

The particle Q makes one complete revolution in 1.8 s, and the normal reaction of the cone on Q has magnitude 0.75 N.

(i) Find the tension in the string. **[2]**

(ii) Find the length of the string. **[5]**

3 Fixed points A and B are 4.8 m apart on the same horizontal level. The midpoint of AB is M. A light elastic string, with natural length 3.9 m and modulus of elasticity 573.3 N, has one end attached to A and the other end attached to B.

(i) Find the elastic energy stored in the string. **[2]**

A particle P is attached to the midpoint of the string, and is released from rest at M. It comes instantaneously to rest when P is 1.8 m vertically below M.

(ii) Show that the mass of P is 15 kg. **[5]**

(iii) Verify that P can rest in equilibrium when it is 1.0 m vertically below M. **[4]**

In general, a light elastic string, with natural length a and modulus of elasticity λ, has its ends attached to fixed points which are a distance d apart on the same horizontal level. A particle of mass m is attached to the midpoint of the string, and in the equilibrium position each half of the string has length h, as shown in Fig. 3.

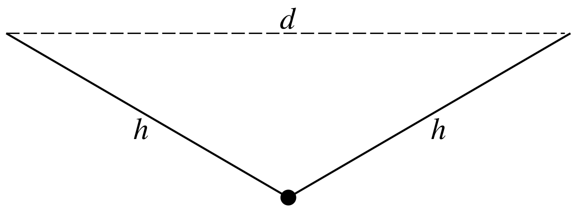

Fig. 3

When the particle makes small oscillations in a vertical line, the period of oscillation is given by the formula

$$\sqrt{\frac{8\pi^2 h^3}{8h^3 - ad^2}}\, m^{\alpha} a^{\beta} \lambda^{\gamma}.$$

(iv) Show that $\frac{8\pi^2 h^3}{8h^3 - ad^2}$ is dimensionless. **[1]**

(v) Use dimensional analysis to find α, β and γ. **[4]**

(vi) Hence find the period when the particle P makes small oscillations in a vertical line centred on the position of equilibrium given in part **(iii)**. **[2]**

4 The region A is bounded by the curve $y = x^2 + 5$ for $0 \leqslant x \leqslant 3$, the x-axis, the y-axis and the line $x = 3$. The region B is bounded by the curve $y = x^2 + 5$ for $0 \leqslant x \leqslant 3$, the y-axis and the line $y = 14$. These regions are shown in Fig. 4.

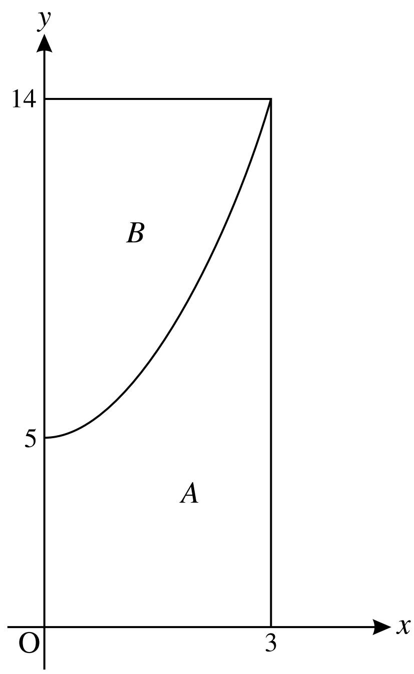

Fig. 4

(i) Find the coordinates of the centre of mass of a uniform lamina occupying the region A. **[9]**

(ii) The region B is rotated through 2π radians about the y-axis to form a uniform solid of revolution R. Find the y-coordinate of the centre of mass of the solid R. **[6]**

(iii) The region A is rotated through 2π radians about the y-axis to form a uniform solid of revolution S. Using your answer to part **(ii)**, or otherwise, find the y-coordinate of the centre of mass of the solid S. **[3]**

Wednesday 25 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4763 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4733270112

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The surface tension of a liquid enables a metal needle to be at rest on the surface of the liquid. The greatest mass m of a needle of length a which can be supported in this way by a liquid of surface tension S is given by

$$m = \frac{2Sa}{g}$$

where g is the acceleration due to gravity.

(i) Show that the dimensions of surface tension are MT^{-2}. **[3]**

The surface tension of water is 0.073 when expressed in SI units (based on kilograms, metres and seconds).

(ii) Find the surface tension of water when expressed in a system of units based on grams, centimetres and minutes. **[3]**

Liquid will rise up a capillary tube to a height h given by $h = \dfrac{2S}{\rho g r}$, where ρ is the density of the liquid and r is the radius of the capillary tube.

(iii) Show that the equation $h = \dfrac{2S}{\rho g r}$ is dimensionally consistent. **[3]**

(iv) Find the radius of a capillary tube in which water will rise to a height of 25 cm. (The density of water is 1000 in SI units.) **[2]**

When liquid is poured onto a horizontal surface, it forms puddles of depth d. You are given that $d = kS^{\alpha}\rho^{\beta}g^{\gamma}$ where k is a dimensionless constant.

(v) Use dimensional analysis to find α, β and γ. **[4]**

Water forms puddles of depth 0.44 cm. Mercury has surface tension 0.487 and density 13 500 in SI units.

(vi) Find the depth of puddles formed by mercury on a horizontal surface. **[3]**

2 A light inextensible string of length 5 m has one end attached to a fixed point A and the other end attached to a particle P of mass 0.72 kg.

At first, P is moving in a vertical circle with centre A and radius 5 m. When P is at the highest point of the circle it has speed $10\,\mathrm{m\,s^{-1}}$.

(i) Find the tension in the string when the speed of P is $15\,\mathrm{m\,s^{-1}}$. **[5]**

The particle P now moves at constant speed in a horizontal circle with radius 1.4 m and centre at the point C which is 4.8 m vertically below A.

(ii) Find the tension in the string. **[3]**

(iii) Find the time taken for P to make one complete revolution. **[4]**

Another light inextensible string, also of length 5 m, now has one end attached to P and the other end attached to the fixed point B which is 9.6 m vertically below A. The particle P then moves with constant speed $7\,\mathrm{m\,s^{-1}}$ in the circle with centre C and radius 1.4 m, as shown in Fig. 2.

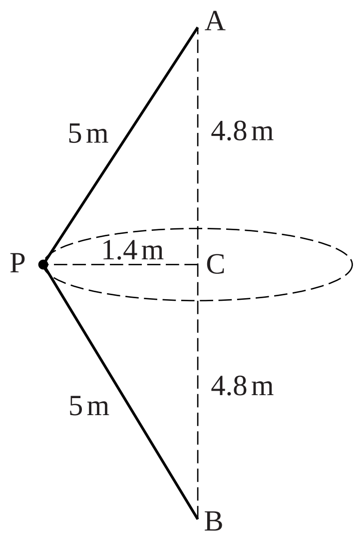

Fig. 2

(iv) Find the tension in the string PA and the tension in the string PB. **[6]**

3 A bungee jumper of mass 75 kg is connected to a fixed point A by a light elastic rope with stiffness $300\,\mathrm{N\,m^{-1}}$. The jumper starts at rest at A and falls vertically. The lowest point reached by the jumper is 40 m vertically below A. Air resistance may be neglected.

(i) Find the natural length of the rope. **[4]**

(ii) Show that, when the rope is stretched and its extension is x metres, $\ddot{x} + 4x = 9.8$. **[3]**

The substitution $y = x - c$, where c is a constant, transforms this equation to $\ddot{y} = -4y$.

(iii) Find c, and state the maximum value of y. **[3]**

(iv) Using standard simple harmonic motion formulae, or otherwise, find

(A) the maximum speed of the jumper,

(B) the maximum deceleration of the jumper. **[3]**

(v) Find the time taken for the jumper to fall from A to the lowest point. **[5]**

4 **(a)** The region T is bounded by the x-axis, the line $y = kx$ for $a \leqslant x \leqslant 3a$, the line $x = a$ and the line $x = 3a$, where k and a are positive constants. A uniform frustum of a cone is formed by rotating T about the x-axis. Find the x-coordinate of the centre of mass of this frustum. **[6]**

(b) A uniform lamina occupies the region (shown in Fig. 4) bounded by the x-axis, the curve $y = 16(1 - x^{-\frac{1}{3}})$ for $1 \leqslant x \leqslant 8$ and the line $x = 8$.

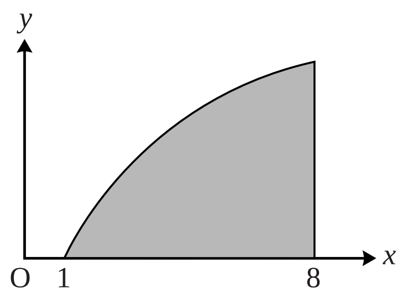

Fig. 4

(i) Find the coordinates of the centre of mass of this lamina. **[8]**

A hole is made in the lamina by cutting out a circular disc of area 5 square units. This causes the centre of mass of the lamina to move to the point (5, 3).

(ii) Find the coordinates of the centre of the hole. **[4]**

OCR
RECOGNISING ACHIEVEMENT

Thursday 31 May 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4763 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 The fixed point A is at a height $4b$ above a smooth horizontal surface, and C is the point on the surface which is vertically below A. A light elastic string, of natural length $3b$ and modulus of elasticity λ, has one end attached to A and the other end attached to a block of mass m. The block is released from rest at a point B on the surface where BC = $3b$, as shown in Fig. 1. You are given that the block remains on the surface and moves along the line BC.

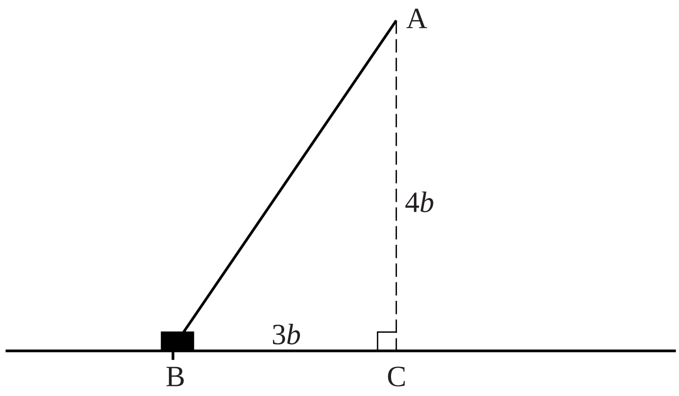

Fig. 1

(i) Show that immediately after release the acceleration of the block is $\frac{2\lambda}{5m}$. **[4]**

(ii) Show that, when the block reaches C, its speed v is given by $v^2 = \frac{\lambda b}{m}$. **[4]**

(iii) Show that the equation $v^2 = \frac{\lambda b}{m}$ is dimensionally consistent. **[3]**

The time taken for the block to move from B to C is given by $km^{\alpha}b^{\beta}\lambda^{\gamma}$, where k is a dimensionless constant.

(iv) Use dimensional analysis to find α, β and γ. **[4]**

When the string has natural length 1.2 m and modulus of elasticity 125 N, and the block has mass 8 kg, the time taken for the block to move from B to C is 0.718 s.

(v) Find the time taken for the block to move from B to C when the string has natural length 9 m and modulus of elasticity 20 N, and the block has mass 75 kg. **[3]**

2 **(a)** Fig. 2 shows a car of mass 800 kg moving at constant speed in a horizontal circle with centre C and radius 45 m, on a road which is banked at an angle of 18° to the horizontal. The forces shown are the weight W of the car, the normal reaction, R, of the road on the car and the frictional force F.

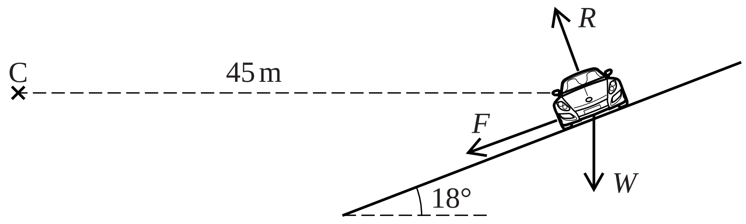

Fig. 2

(i) Given that the frictional force is zero, find the speed of the car. **[4]**

(ii) Given instead that the speed of the car is $15\,\mathrm{m\,s^{-1}}$, find the frictional force and the normal reaction. **[7]**

(b) One end of a light inextensible string is attached to a fixed point O, and the other end is attached to a particle P of mass m kg. Starting with the string taut and P vertically below O, P is set in motion with a horizontal velocity of $7\,\mathrm{m\,s^{-1}}$. It then moves in part of a vertical circle with centre O. The string becomes slack when the speed of P is $2.8\,\mathrm{m\,s^{-1}}$.

Find the length of the string. Find also the angle that OP makes with the upward vertical at the instant when the string becomes slack. **[7]**

3 A particle Q is performing simple harmonic motion in a vertical line. Its height, x metres, above a fixed level at time t seconds is given by

$$x = c + A\cos(\omega t - \phi)$$

where c, A, ω and ϕ are constants.

(i) Show that $\ddot{x} = -\omega^2(x - c)$. **[3]**

Fig. 3 shows the displacement-time graph of Q for $0 \leqslant t \leqslant 14$.

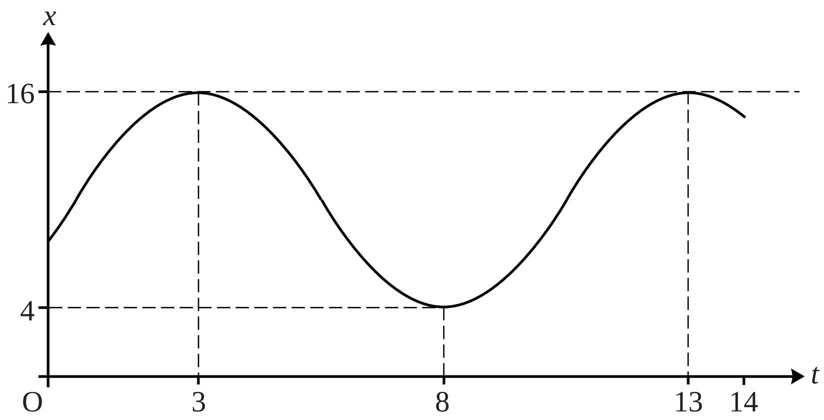

Fig. 3

(ii) Find exact values for c, A, ω and ϕ. **[6]**

(iii) Find the maximum speed of Q. **[2]**

(iv) Find the height and the velocity of Q when $t = 0$. **[3]**

(v) Find the distance travelled by Q between $t = 0$ and $t = 14$. **[4]**

4 **(a)** A uniform lamina occupies the region bounded by the x-axis, the y-axis and the curve $y = 3 - \sqrt{x}$ for $0 \leqslant x \leqslant 9$. Find the coordinates of the centre of mass of this lamina. **[9]**

(b) Fig. 4.1 shows the region bounded by the line $x = 2$ and the part of the circle $y^2 = 25 - x^2$ for which $2 \leqslant x \leqslant 5$. This region is rotated about the x-axis to form a uniform solid of revolution S.

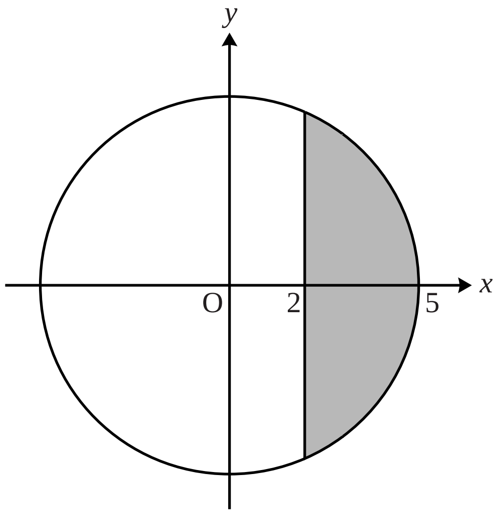

Fig. 4.1

(i) Find the x-coordinate of the centre of mass of S. **[5]**

The solid S rests in equilibrium with its curved surface in contact with a rough plane inclined at 25° to the horizontal. Fig. 4.2 shows a vertical section containing AB, which is a diameter and also a line of greatest slope of the flat surface of S. This section also contains XY, which is a line of greatest slope of the plane.

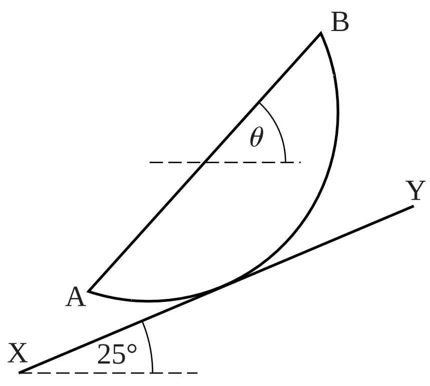

Fig. 4.2

(ii) Find the angle θ that AB makes with the horizontal. **[4]**

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OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
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- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 **(a)** A particle P is executing simple harmonic motion, and the centre of the oscillations is at the point O. The maximum speed of P during the motion is $5.1\,\mathrm{m\,s^{-1}}$. When P is 6 m from O, its speed is $4.5\,\mathrm{m\,s^{-1}}$. Find the period and the amplitude of the motion. **[6]**

(b) The force F of gravitational attraction between two objects of masses m_1 and m_2 at a distance d apart is given by $F = \dfrac{Gm_1m_2}{d^2}$, where G is the universal gravitational constant.

(i) Find the dimensions of G. **[3]**

Three objects, each of mass m, are moving in deep space under mutual gravitational attraction. They move round a single circle with constant angular speed ω, and are always at the three vertices of an equilateral triangle of side R. You are given that $\omega = kG^{\alpha}m^{\beta}R^{\gamma}$, where k is a dimensionless constant.

(ii) Find α, β and γ. **[5]**

For three objects of mass 2500 kg at the vertices of an equilateral triangle of side 50 m, the angular speed is $2.0 \times 10^{-6}\,\mathrm{rad\,s^{-1}}$.

(iii) Find the angular speed for three objects of mass 4.86×10^{14} kg at the vertices of an equilateral triangle of side 30 000 m. **[4]**

2 **(a)** A fixed solid sphere with a smooth surface has centre O and radius 0.8 m. A particle P is given a horizontal velocity of $1.2\,\mathrm{m\,s^{-1}}$ at the highest point on the sphere, and it moves on the surface of the sphere in part of a vertical circle of radius 0.8 m.

(i) Find the radial and tangential components of the acceleration of P at the instant when OP makes an angle $\frac{1}{6}\pi$ radians with the upward vertical. (You may assume that P is still in contact with the sphere.) **[5]**

(ii) Find the speed of P at the instant when it leaves the surface of the sphere. **[6]**

(b) Two fixed points R and S are 2.5 m apart with S vertically below R. A particle Q of mass 0.9 kg is connected to R and to S by two light inextensible strings; Q is moving in a horizontal circle at a constant speed of $5\,\mathrm{m\,s^{-1}}$ with both strings taut. The radius of the circle is 2.4 m and the centre C of the circle is 0.7 m vertically below S, as shown in Fig. 2.

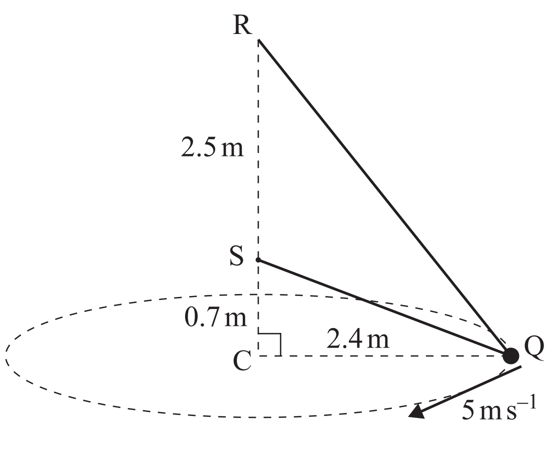

Fig. 2

Find the tension in the string RQ and the tension in the string SQ. **[7]**

3 Two fixed points X and Y are 14.4 m apart and XY is horizontal. The midpoint of XY is M. A particle P is connected to X and to Y by two light elastic strings. Each string has natural length 6.4 m and modulus of elasticity 728 N. The particle P is in equilibrium when it is 3 m vertically below M, as shown in Fig. 3.

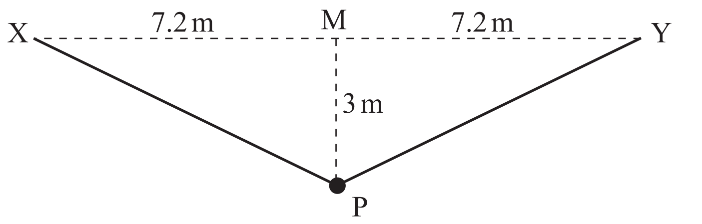

Fig. 3

(i) Find the tension in each string when P is in the equilibrium position. **[3]**

(ii) Show that the mass of P is 12.5 kg. **[3]**

The particle P is released from rest at M, and moves in a vertical line.

(iii) Find the acceleration of P when it is 2.1 m vertically below M. **[5]**

(iv) Explain why the maximum speed of P occurs at the equilibrium position. **[1]**

(v) Find the maximum speed of P. **[6]**

4 **(a)** The region enclosed between the curve $y = x^4$ and the line $y = h$ (where h is positive) is rotated about the y-axis to form a uniform solid of revolution. Find the y-coordinate of the centre of mass of this solid. **[5]**

(b) The region A is bounded by the x-axis, the curve $y = x + \sqrt{x}$ for $0 \leqslant x \leqslant 4$, and the line $x = 4$. The region B is bounded by the y-axis, the curve $y = x + \sqrt{x}$ for $0 \leqslant x \leqslant 4$, and the line $y = 6$. These regions are shown in Fig. 4.

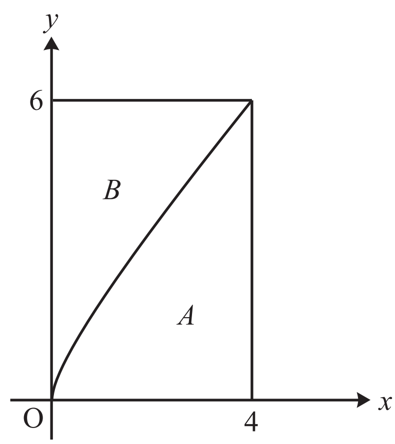

Fig. 4

(i) A uniform lamina occupies the region A. Show that the x-coordinate of the centre of mass of this lamina is 2.56, and find the y-coordinate. **[9]**

(ii) Using your answer to part **(i)**, or otherwise, find the coordinates of the centre of mass of a uniform lamina occupying the region B. **[4]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

OCR RECOGNISING ACHIEVEMENT

Thursday 6 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715810613

INSTRUCTIONS TO CANDIDATES

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- Answer **all** the questions.
- Do **not** write in the bar codes.
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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 **(a)** A particle P of mass 1.5 kg is connected to a fixed point by a light inextensible string of length 3.2 m. The particle P is moving as a conical pendulum in a horizontal circle at a constant angular speed of 2.5 rad s^{-1}.

(i) Find the tension in the string. **[4]**

(ii) Find the angle that the string makes with the vertical. **[2]**

(b) A particle Q of mass m moves on a smooth horizontal surface, and is connected to a fixed point on the surface by a light elastic string of natural length d and stiffness k. With the string at its natural length, Q is set in motion with initial speed u perpendicular to the string. In the subsequent motion, the maximum length of the string is $2d$, and the string first returns to its natural length after time t.

You are given that $u = \sqrt{\frac{4kd^2}{3m}}$ and $t = Ak^{\alpha}d^{\beta}m^{\gamma}$, where A is a dimensionless constant.

(i) Show that the dimensions of k are MT^{-2}. **[1]**

(ii) Show that the equation $u = \sqrt{\frac{4kd^2}{3m}}$ is dimensionally consistent. **[2]**

(iii) Find α, β and γ. **[4]**

You are now given that Q has mass 5 kg, and the string has natural length 0.7 m and stiffness 60 N m^{-1}.

(iv) Find the initial speed u, and use conservation of energy to find the speed of Q at the instant when the length of the string is double its natural length. **[5]**

2 A particle P of mass 0.25 kg is connected to a fixed point O by a light inextensible string of length a metres, and is moving in a vertical circle with centre O and radius a metres. When P is vertically below O, its speed is $8.4\,\mathrm{m\,s^{-1}}$. When OP makes an angle θ with the downward vertical, and the string is still taut, P has speed $v\,\mathrm{m\,s^{-1}}$ and the tension in the string is T N, as shown in Fig. 2.

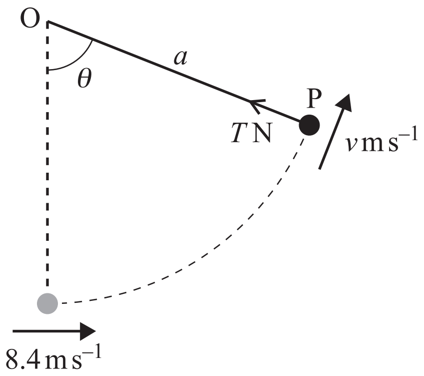

Fig. 2

(i) Find an expression for v^2 in terms of a and θ, and show that

$$T = \frac{17.64}{a} + 7.35\cos\theta - 4.9.$$ **[7]**

(ii) Given that $a = 0.9$, show that P moves in a complete circle. Find the maximum and minimum magnitudes of the tension in the string. **[4]**

(iii) Find the largest value of a for which P moves in a complete circle. **[3]**

(iv) Given that $a = 1.6$, find the speed of P at the instant when the string first becomes slack. **[4]**

3 A light spring, with modulus of elasticity 686 N, has one end attached to a fixed point A. The other end is attached to a particle P of mass 18 kg which hangs in equilibrium when it is 2.2 m vertically below A.

(i) Find the natural length of the spring AP. **[2]**

Another light spring has natural length 2.5 m and modulus of elasticity 145 N. One end of this spring is now attached to the particle P, and the other end is attached to a fixed point B which is 2.5 m vertically below P (so leaving the equilibrium position of P unchanged). While in its equilibrium position, P is set in motion with initial velocity $3.4\,\mathrm{m\,s^{-1}}$ vertically downwards, as shown in Fig. 3. It now executes simple harmonic motion along part of the vertical line AB.

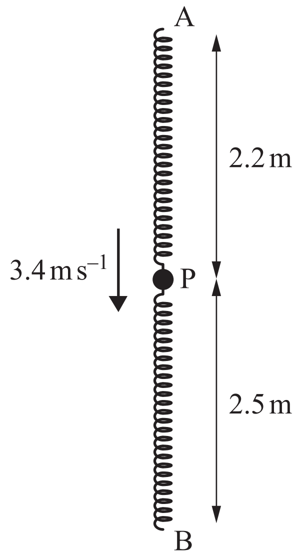

Fig. 3

At time t seconds after it is set in motion, P is x metres below its equilibrium position.

(ii) Show that the tension in the spring AP is $(176.4 + 392x)$N, and write down an expression for the thrust in the spring BP. **[3]**

(iii) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -25x$. **[3]**

(iv) Find the period and the amplitude of the motion. **[3]**

(v) Find the magnitude and direction of the velocity of P when $t = 2.4$. **[3]**

(vi) Find the total distance travelled by P during the first 2.4 seconds of its motion. **[4]**

4 **(a)** A uniform solid of revolution S is formed by rotating the region enclosed between the x-axis and the curve $y = x\sqrt{4-x}$ for $0 \leqslant x \leqslant 4$ through 2π radians about the x-axis, as shown in Fig. 4.1. O is the origin and the end A of the solid is at the point $(4, 0)$.

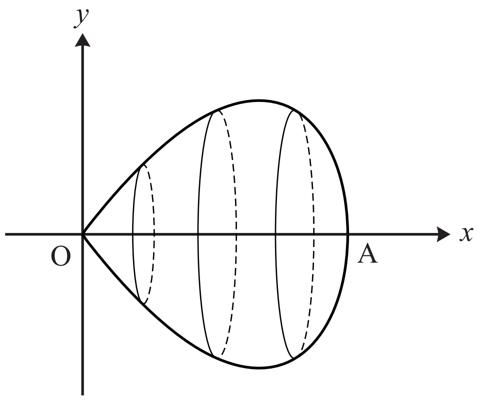

Fig. 4.1

(i) Find the x-coordinate of the centre of mass of the solid S. **[6]**

The solid S has weight W, and it is freely hinged to a fixed point at O. A horizontal force, of magnitude W acting in the vertical plane containing OA, is applied at the point A, as shown in Fig. 4.2. S is in equilibrium.

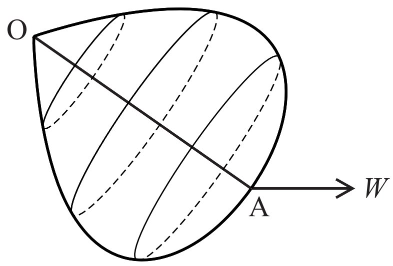

Fig. 4.2

(ii) Find the angle that OA makes with the vertical. **[3]**

[Question 4(b) is printed overleaf]

(b) Fig. 4.3 shows the region bounded by the x-axis, the y-axis, the line $y = 8$ and the curve $y = (x-2)^3$ for $0 \leqslant y \leqslant 8$.

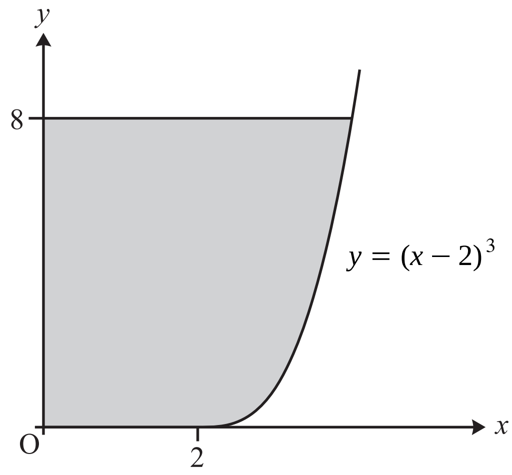

Fig. 4.3

Find the coordinates of the centre of mass of a uniform lamina occupying this region. **[9]**

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